Background Information

The study of individual differences has a history of investigating mathematics abilities. Recently, research has provided evidence of two specific variables within tests of mathematics skills: (a) Computation, which involves knowledge and skill in applying basic mathematics concepts of addition, subtraction, multiplication, and division to simple problems (Fuchs et al., 1994; Jiban & Deno, 2007) and (b) Concepts and Applications, which involves knowledge and skills in applying math concepts to more complex problems including word problems (Fuchs et al., 2008).

Previous research (e.g., Thibler, Shinn, & Smolkowski, 2002) suggested a two-factor model of mathematics with computation and concepts and applications as distinct, but related constructs. Important questions remain regarding the degree to which other constructs, such as processing speed and reading, introduce construct-irrelevant variance when measuring computation and concepts and applications via brief, fluency-based curriculum-based measurement (CBM) instruments.

Purpose

This study investigated the factor structure of brief assessments of mathematics skills, including assessments of mathematics accuracy and fluency from CBM measures as well as from norm-referenced achievement batteries. Because reading is required during completion of many concepts and applications tests, models examining the effects of reading skills on concepts and applications measures were examined. Because CBM measures and other math measures are typically fluency-based, models examining their relations with content-neutral processing speed tests were examined.

Method

Participants

Participants in this study were 190 students in grades 2 and 3 at a university-based public school.

Measures

Measures are listed in Table 1.

Procedure

A total of 14 measures were administered during the school day over two consecutive days for each classroom in the last month of the school year. For CBM measures, Grade 2 and 3 participants completed Grade 3 probes.

To control for order effects, the order in which the tests were presented was counterbalanced across classrooms and days with 5 math tests, 1 reading test, and 1 processing speed test administered each day.

Analyses

Data obtained from the measures was submitted to a correlational analysis, and then, a confirmatory factor analysis was conducted. A priori specified models were modeled and estimated using the Amos 18. Missing data and free parameters were estimated using maximum-likelihood estimation, and χ² difference tests were conducted to determine the appropriateness of releasing or imposing any restriction on the parameters. Parameters were determined to be statistically significant based on an a priori alpha level of .05.

Modeling

Model 1: Single-factor model with one factor on which all indicators loaded

Model 2: Two-factor model with Academic Achievement (including all reading and math measures as indicators) and Processing Speed as factors

Model 3: Three-factor model with Math, Reading, and Processing Speed as factors

Model 4: Four-factor model with Computation, Concepts and Applications, Reading, and Processing Speed as factors

Model 5: Bi-factor model with Computation and Concepts and Applications (as in Model 4) and a Timed factor with both processing speed and all fluency-based measures as indicators.

Results and Discussion

Table 2. Model Fit Statistics

<table>
<thead>
<tr>
<th>Model</th>
<th>Test</th>
<th>df</th>
<th>χ²</th>
<th>df df</th>
<th>mol Comp</th>
<th>mol Difference</th>
<th>mol Difference</th>
<th>CFI</th>
<th>TLI</th>
<th>RMSEA</th>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>.83</td>
<td>.77</td>
<td>.15</td>
<td>136.16</td>
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<td>272.55</td>
<td>76</td>
<td>Model 2 to Model 1</td>
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<td>-1</td>
<td>.84</td>
<td>.75</td>
<td>.15</td>
<td>136.16</td>
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<td>Model 3</td>
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<td>74</td>
<td>Model 3 to Model 2</td>
<td>- .50</td>
<td>-2</td>
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<td>.73</td>
<td>.12</td>
<td>150.14</td>
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<td>.81</td>
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<td>-</td>
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<td>.84</td>
<td>.13</td>
<td>111.36</td>
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</table>

Results

The 4-factor Model 4 proved to be the best-fitting model for the data, indicating that Computation, Concepts and Applications, Reading, and Processing Speed are distinct, but related constructs. As evident in Figure 1, Reading was significantly correlated with both Computation and Concepts and Applications, but was more highly correlated with the Concepts and Applications factor. This is not unexpected as Concepts and Applications test items include words in addition to numbers. Processing Speed was also significantly correlated with all three remaining factors, but was more highly correlated with Computation and Concepts and Applications than with Reading tests.

Implications

Results supported previous models of mathematics ability that include Computation and Concepts and Applications as distinct, but related constructs.

• Reading and Processing Speed were shown to be distinct from but significantly correlated with both Computation and Concepts and Applications.

• Concepts and Applications tests did not have significant loadings on the Reading factor, suggesting Reading does not contribute construct-irrelevant variance on these tests.

• Processing Speed was shown to contribute statistically significant construct-irrelevant variance on performance on all fluency-based measures.

• Decisions regarding educational programming are often influenced by scores from the type of tests used in the current study. Careful consideration of the effects of the broad Processing Speed ability should be taken into account when planning for instruction based on student scores from similar measures.